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ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains exercises on number plane jumping rules, a summary of the problems in the film "Jumping Rules in the Plane, Part I," and the supplement. (MK)

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THE ARITHMETIC PROJECT COURSE FOR TEACHERS

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TOPIC: Number Plane Jumping Rules.

FILM: Jumping Rules in the Plane, Part One, Grade 6

SUPPLEMENT: Composing Number Line Rules to Move Two Points to Two Points

NAME: .....

18

SE 031 118

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BOOK EIGHTEEN

NUMBER PLANE JUMPING RULES

I.

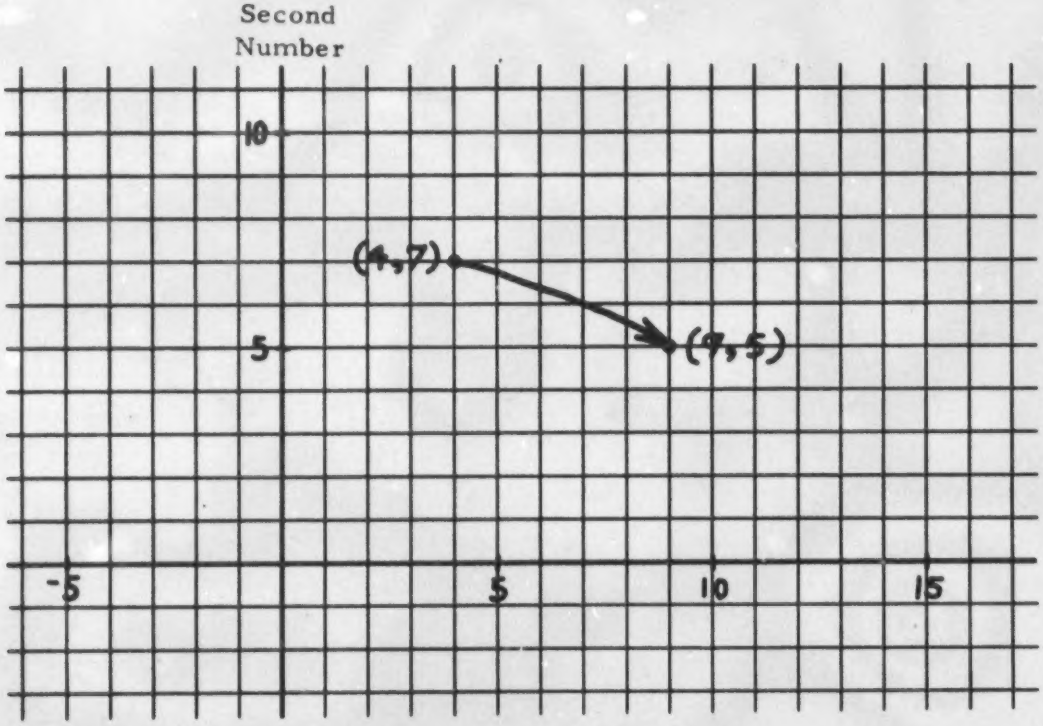
You have seen jumping rules for a number line. Jumping rules for a number plane require frames of two shapes. Here is an example:

$$(\square, \triangle) \longrightarrow (\square + 5, \triangle - 2)$$

If your starting point is (4, 7), your jump is

$$(\boxed{4}, \overset{7}{\triangle}) \longrightarrow (\boxed{4} + 5, \overset{7}{\triangle} - 2)$$

and you land at (9, 5). The jump can be shown like this:

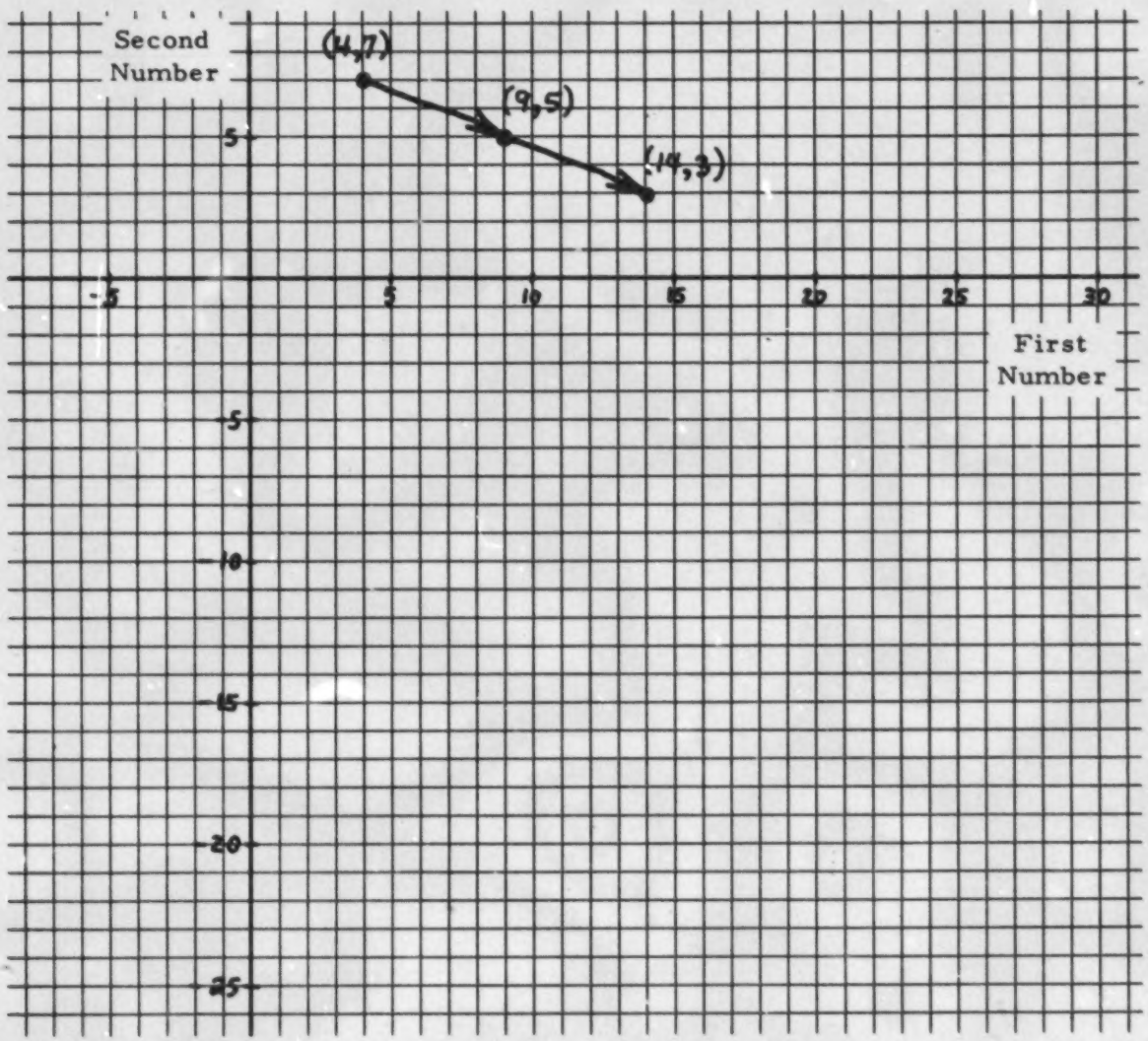


Continue with the rule: $(\square, \triangle) \longrightarrow (\square + 5, \triangle - 2)$

1. With the same rule, make another jump from (9,5). Land: (14,3)
2. Start at (0,0) and make one jump with the same rule. Land: _____
3. Start at (0,0) and make 6 consecutive jumps. Your 6th jump lands on: _____
4. Where do you have to start so that after 11 jumps you land at (0,0)?

   _____
5. Think of the line through (0,-26) and (117,-26). If you start at (4,7) and make consecutive jumps with this rule, you will eventually cross (not land on) this horizontal line. How many jumps do you need?

   _____



☆6. Rules:

$(\square, \triangle) \xrightarrow{a} (\square - 2, \triangle)$

$(\square, \triangle) \xrightarrow{b} (\square, \triangle - 2)$

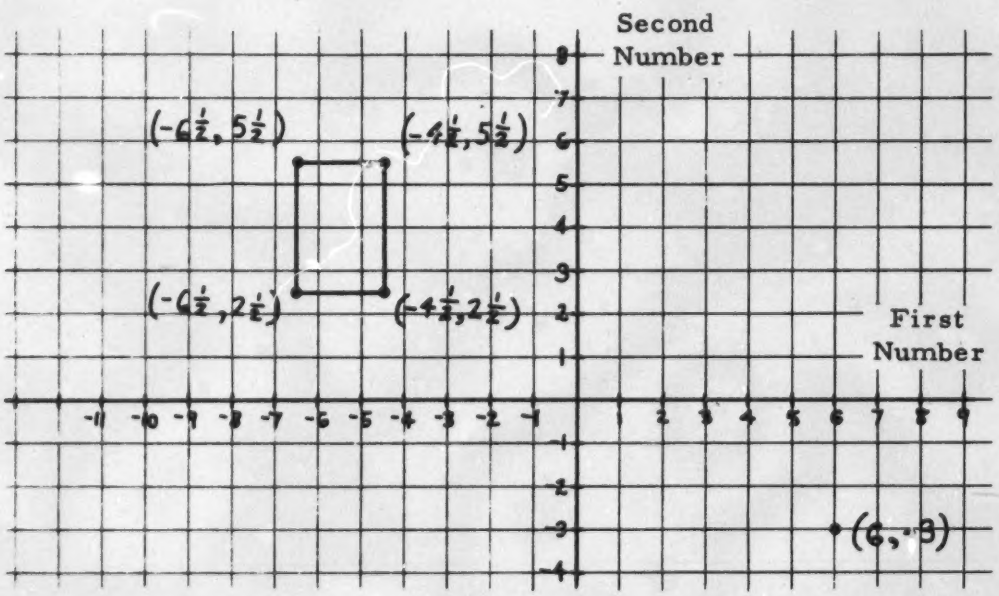
$(\square, \triangle) \xrightarrow{c} (\square + 3, \triangle + 3)$

$(\square, \triangle) \xrightarrow{d} (\triangle, \square)$

(a) You may make successive jumps with any combination of the rules above. What is the smallest number of jumps that will take you from (6, -3) to some point within the rectangle outlined below? Specify the jumps by listing the points and the rules used, such as

$$(6, -3) \xrightarrow{b} (6, -5) \xrightarrow{c} (9, -2)$$

_____ Number of jumps: _____

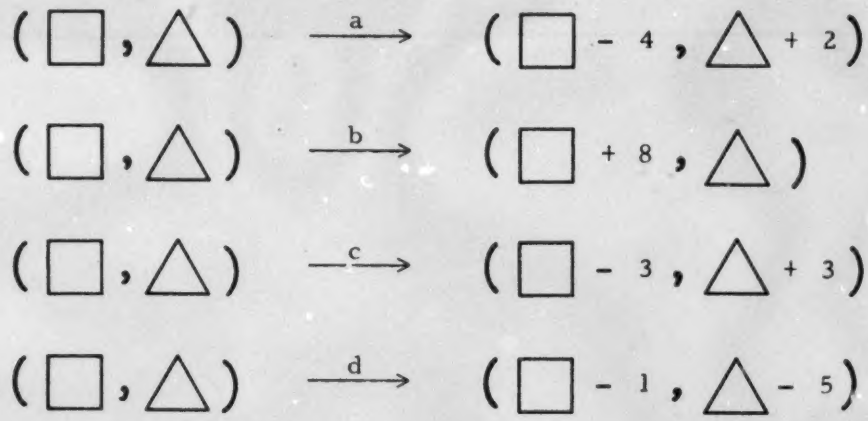


(b) Starting at (6, -3), do you think you could land at $(\frac{1}{4}, \frac{1}{2})$? _____

(c) No computing. Starting at (6, -3), do you think you could land at (3,089 , 187)? _____ Comment:

In doing the exercises that follow, use the graph paper at the end of this lesson to plot jumps when you are in doubt about what a rule does.

II. Consider the following number plane jumping rules:



Do the following. (The first has been done for you.)

1. (20, 30) $\xrightarrow{ab}$ (24,32)

2. (2, 5) $\xrightarrow{abcd}$ _____

3. (0, -4) $\xrightarrow{abcd}$ _____

4. (-10, -10) $\xrightarrow{abcd}$ _____

5. (0, 0) $\xrightarrow{abcd}$ _____

Do the following without computation:

6. (100, 75) $\xrightarrow{aabbccdd}$ _____

7. (100, 75) $\xrightarrow{addcbcba}$ _____

8. (50, 50) $\xrightarrow{aaabbbcccdd}$ _____ (Careful!)

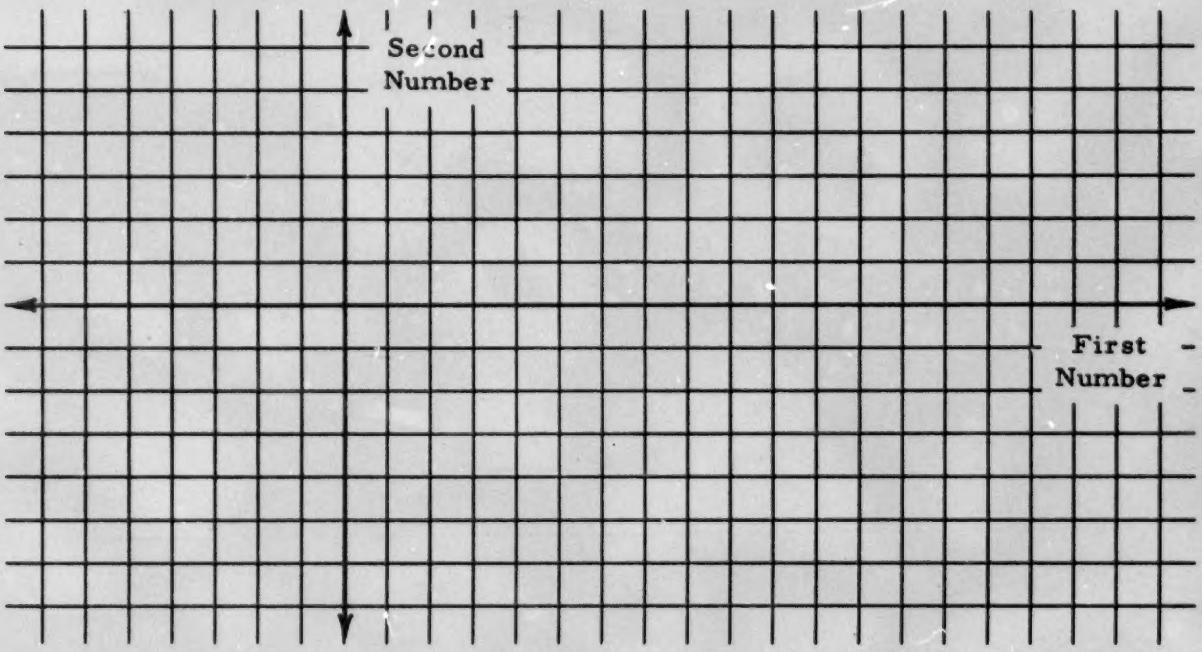
III. Jumping rules:

$(\square, \triangle) \xrightarrow{A} (\square + 3, \triangle - 4)$

$(\square, \triangle) \xrightarrow{B} (\square - 6, \triangle)$

$(\square, \triangle) \xrightarrow{C} (\square, \triangle + 8)$

1. Start at (10 , 4) and make a jump with rule A , another jump with rule A , a jump with rule B , and a jump with rule C . Plot the landing points below, connect successive points with straight lines, and find the area of the resulting figure.



Area: _____

2. Think of using the same rules in the same order, starting at (100 , 40), and of drawing the corresponding picture. Will the new area be more, less, or the same as the previous area? _____

Comment:

IV. Jumping rule:

$$(\square, \triangle) \longrightarrow (\square - \triangle, \square - \triangle)$$

1. Start at (6 , 5) and make one jump. Land? _____

2. Start at (7 , 3) and make <u>two</u> <u>consecutive</u> jumps. Land? _____

3. Start at $(-4\frac{1}{4}, 3)$ and make one jump. Land? _____

Make another jump. Land? _____

Comment:

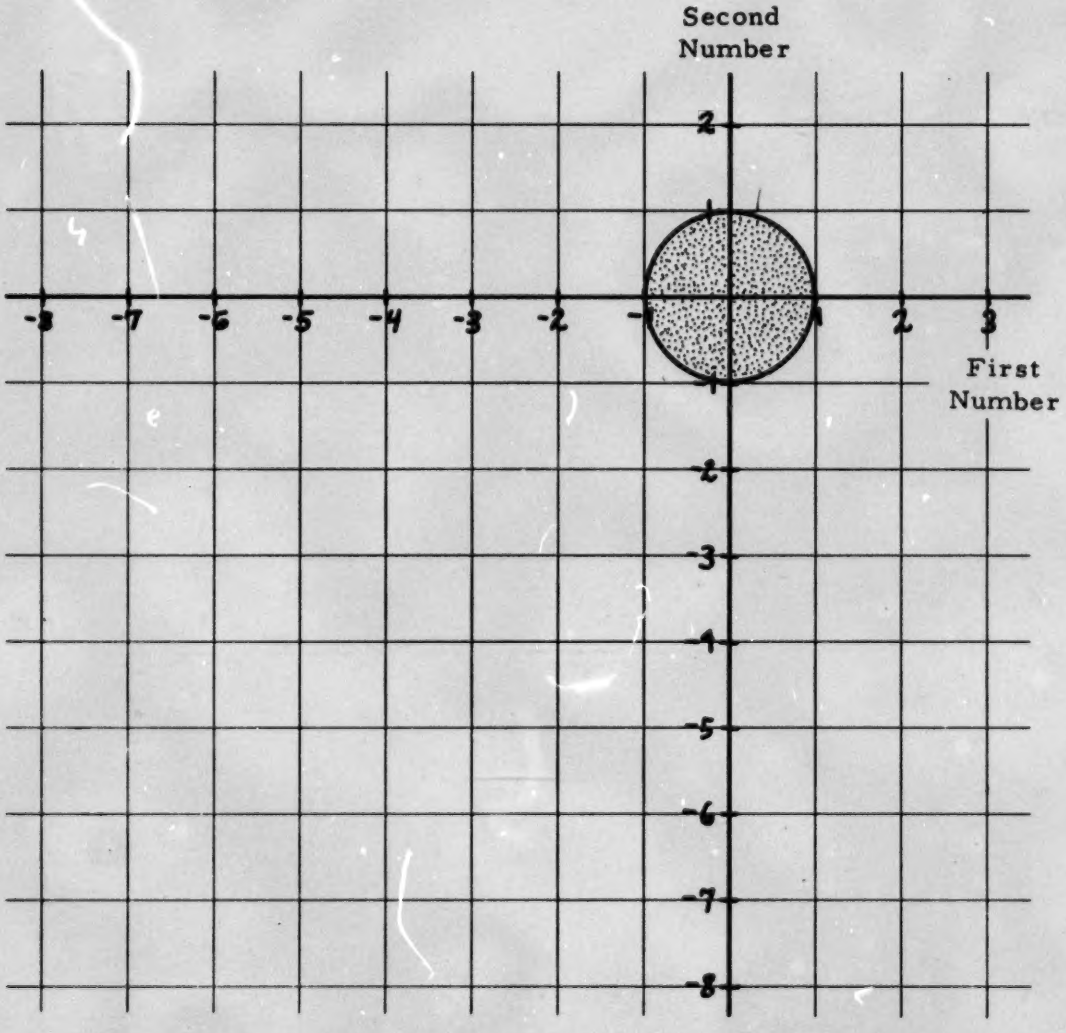
V. Jumping rule:

$$(\square, \triangle) \longrightarrow (\square + \triangle, \square + \triangle)$$

1. Start at (12, 7) and make a jump. Land? _____ Make another jump from there. Land? _____ What can you say about the next 100 jumps?

2. Give a starting point within or on the edge of the shaded disc shown below so that eventually successive jumps will take you to (-8, -8).

Starting point: _____



Continue with the rule: $(\square, \triangle) \longrightarrow (\square + \triangle, \square + \triangle)$

3. Give all standstill points:

_____

Prove that you have them all:

4. Give a point other than a standstill point from which a jump lands on a standstill point: _____

5. Is there a starting point so that in one jump you land at $(10, -9\frac{1}{2})$?

_____ At $(10, 9\frac{1}{2})$? _____ At $(10, -10)$? _____

At $(6\frac{1}{2}, 6\frac{1}{2})$? _____

Explain:

6. Without computing, do you think there is a starting point such that after 43 successive jumps you land at $(1{,}017\frac{1}{9}, 1{,}017\frac{1}{9})$? _____

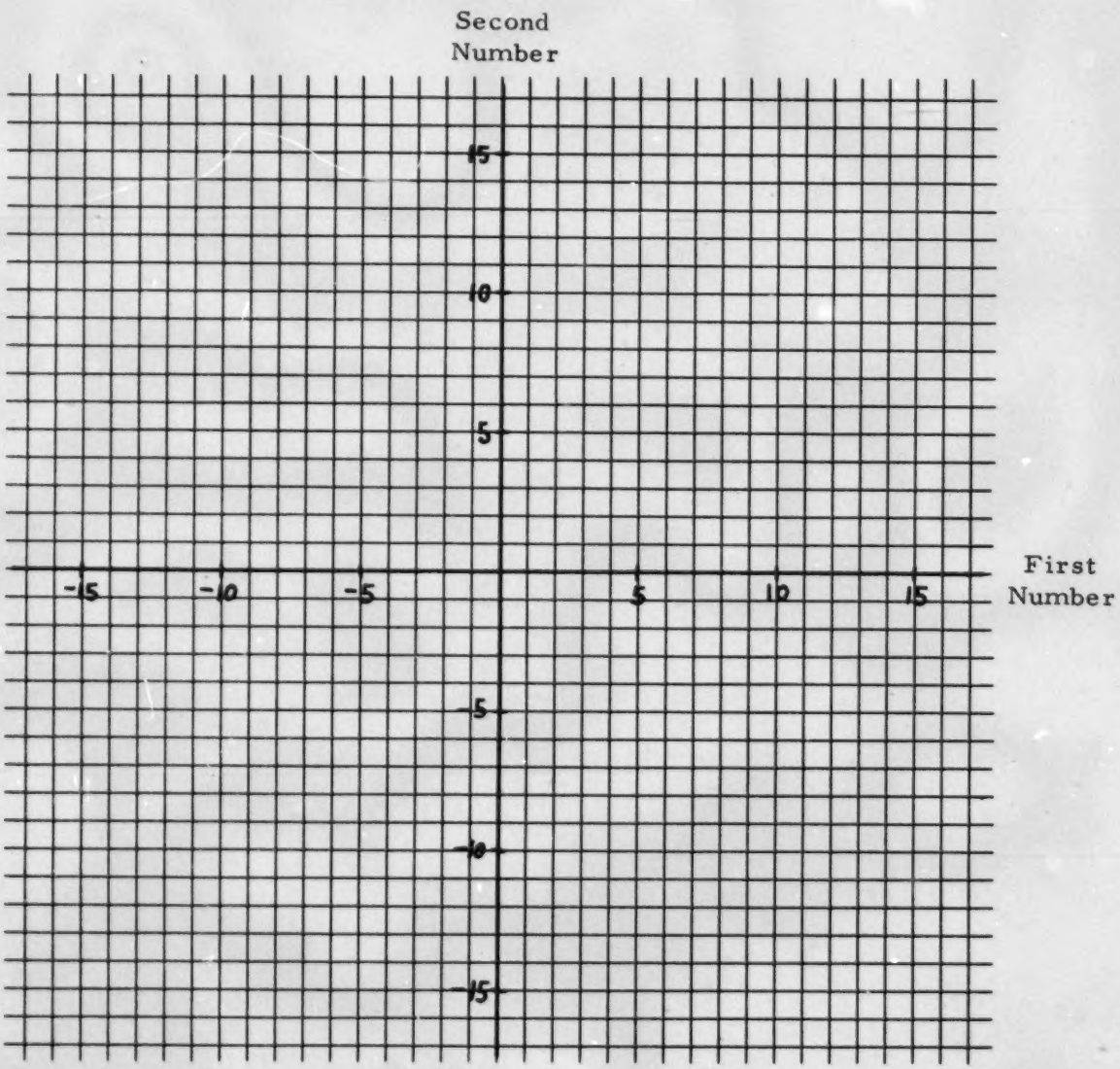
Explain:

Continue with the rule: $(\square, \triangle) \longrightarrow (\square + \triangle, \square + \triangle)$

7. Give five points each of which when used as a starting point takes you to (9, 9) in one jump.

(, $2\frac{1}{3}$) (,) (,) (-2 ,) (, -5)

On the graph below, show all points from which one jump takes you to (9, 9). (Hint: Start by plotting the points you gave above.)

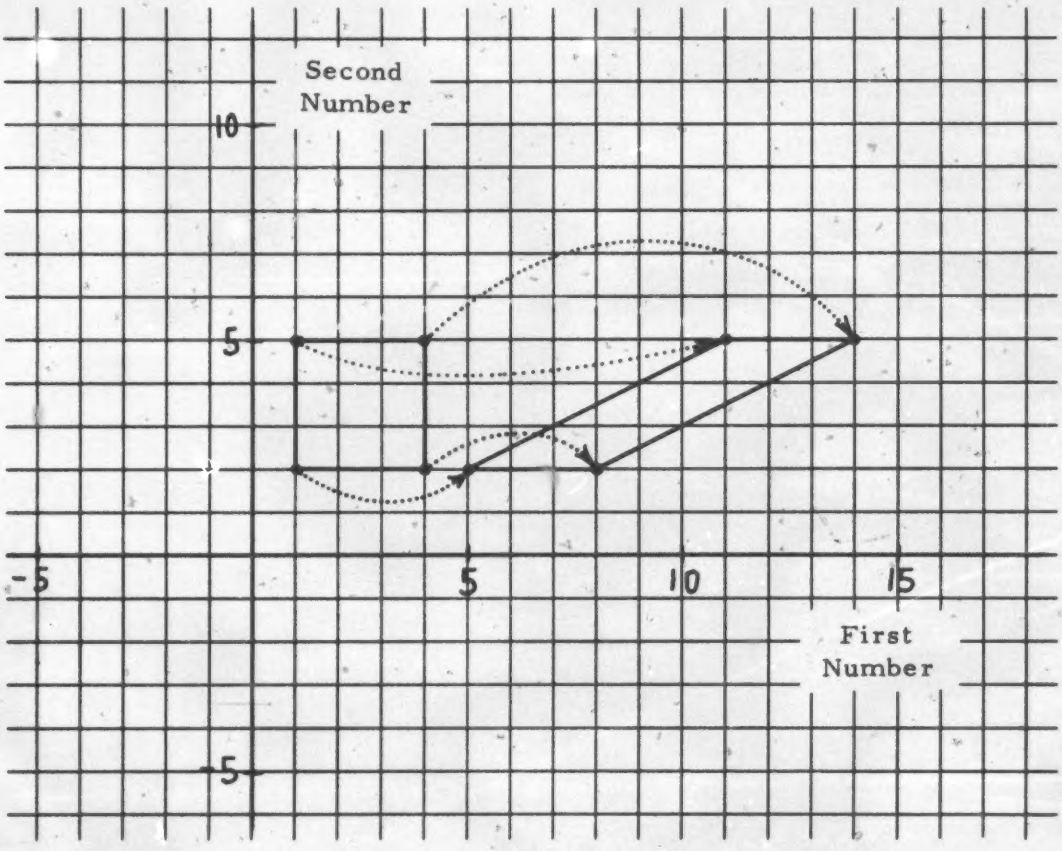


Continue with the rule: $(\square, \triangle) \longrightarrow (\square + \triangle, \square + \triangle)$

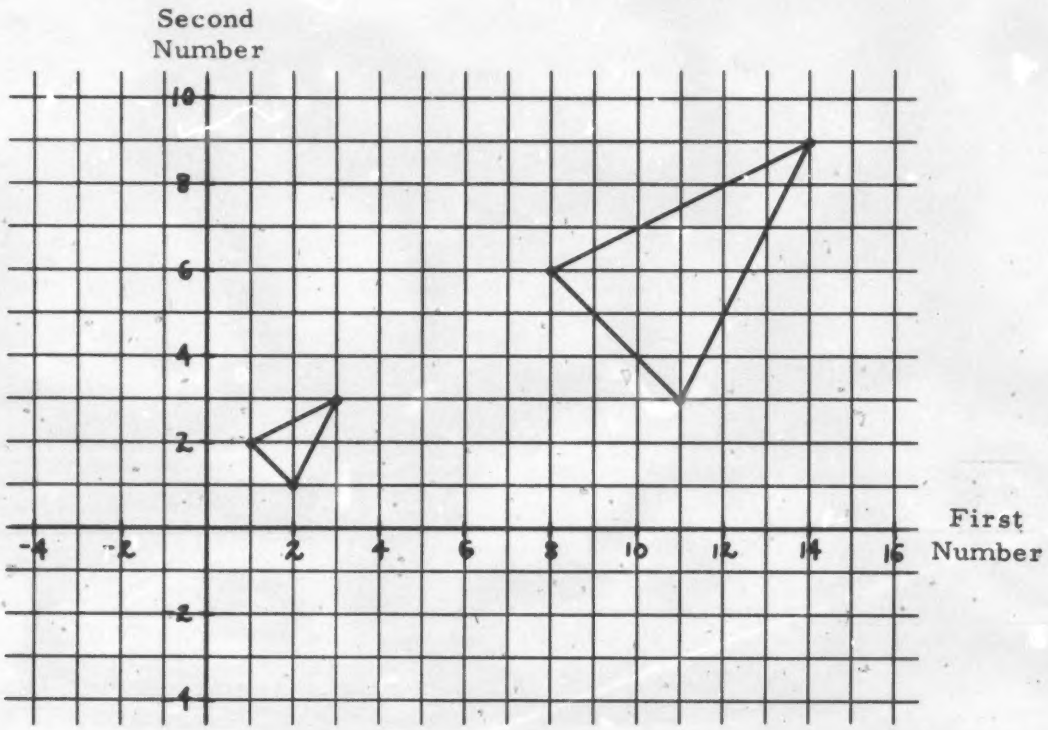
☆8. Describe the graph of all points that would eventually lead to (9,9) as a landing point after one or more jumps. (Show your answer on the preceding page if you wish.)

VI.

The square below has vertices (1,2), (1,5), (4,5) and (4,2). If you use the jumping rule $(\square, \triangle) \longrightarrow (\square + 2\times\triangle, \triangle)$ for each point on the square, you obtain the parallelogram with vertices (5,2), (11,5), (14,5) and (8,2).

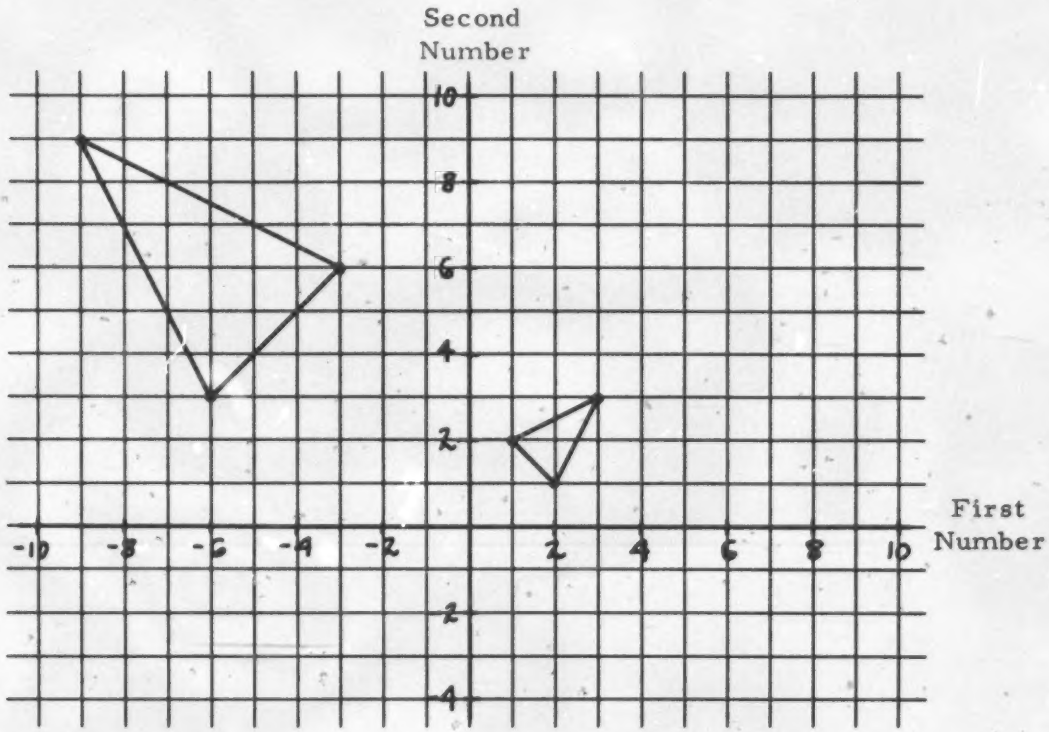


☆1. Find a rule that will transform the small triangle below into the larger triangle. In other words, your rule should be such that when you use the points of the small triangle as starting points, you get the points of the large triangle as landing points.



Rule: (□ , △) ⟶ _____

☆2. Find a rule that will transform the small triangle into the larger triangle. As in the previous problem, when you use your rule on the points of the small triangle, you should get the points of the larger triangle as landing points.



Rule: (□ , △) ⟶ _____

Summary of Problems in the Film
"Jumping Rules in the Plane, Part I"

6th Grade, James Russell Lowell School, Watertown, Massachusetts
Teacher: Lee Osburn

Just as you can have jumping rules on a number line, you can have jumping rules on a plane.

How many numbers does it take to name any point up here?

(2)

So our starting points will have two numbers. We'll use this rule:
$(\square , \triangle) \longrightarrow (\square + 3 , \triangle + 5)$. Someone pick a point.

(5 , 4)

If you start at (5 , 4) with this rule, where do you land?

(8 , 9)

Pick another point. (3 , 5)

Where do you land? (6 , 10)

Start at (0 , 0) and use this rule twice. Where do you land?

(6 , 10)

Start at (0 , 0) and use this rule one hundred times. What point would you land on?

(300 , 500)

In what direction are you moving with this rule if you start anywhere up here?

(Up and to the right.)

What direction do you think you'll move with this rule:
$(\square , \triangle) \longrightarrow (\square - 7 , \triangle + 5)$?

What would I have to do to the rule to change it so that I could move down and to the left?

"Make both of them subtraction:
$(\square , \triangle) \longrightarrow (\square - 7 , \triangle - 5)$."

What could I do to this rule to change it so that I could move only to the right and not up or down at all?

"Have a plus sign in front of the 7 and have a 0 for the 5."

Teacher writes $(\square , \triangle) \longrightarrow (\square + 7 , \triangle - 0)$. Now make up a rule that will move only down.

"The first one take away 0 and the other one add 7."

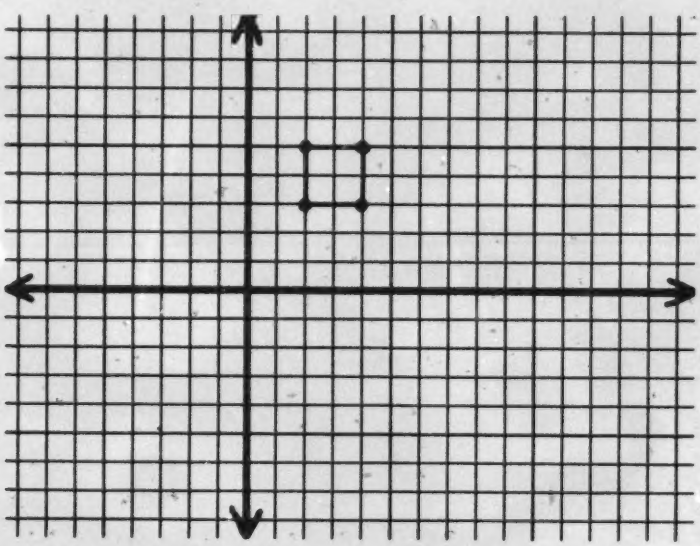
Writes $(\square , \triangle) \longrightarrow (\square - 0 , \triangle + 7)$. What do you think this rule is going to do?

"It's going to go up."

Change it so that it will go down.

"Put a subtraction sign instead of an adding sign before the 7."

Teacher puts a square on the graphboard.



What is the area of that square? (4)

What is the name of that point there? (4 , 3)

How about that one? (4 , 5)

And that one? (2 , 5)

And that one? (2 , 3)

We are going to take all of the points of that square and put them through some rule. Using the rule $(\square , \triangle) \longrightarrow (\square + 9 , \triangle)$, is the area going to get bigger or smaller? Or is it going to stay the same? Is (2 , 4) a point on that square? Where does it go?

(11 , 4)

Pick some other points on that square and see where they go.

(3 , 3) ⟶ (12 , 3)

(4 , 4) ⟶ (13 , 4)

(2 , 5) ⟶ (11 , 5)

(4 , 3) ⟶ (13 , 3)

What do you think is going to happen to all the points in here if we run them all through the rule?

"They are going to form a box." ')

What happened to the area? "It stayed the same."

What rule could I use so that we could take that square and move it down here someplace, but keep the area the same?

I can add or subtract and it won't change anything. Give a rule that will change the area. (Class doesn't think it can be done.)

New rule: (□ , △) ⟶ (□ × 5 , △). Will the area change or stay the same now? Teacher changes rule to (□ , △) ⟶ (□ × 3 , △).

Try (2 , 5). (6 , 5)

Try one of the other points up there. (2 , 3) goes to (6 , 3)

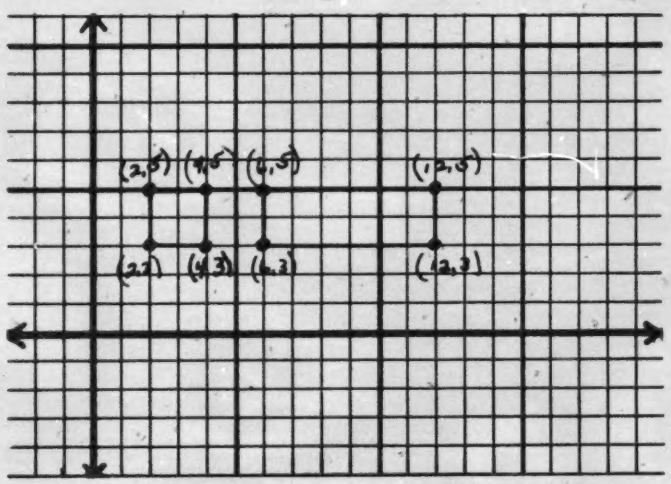
Maybe you are right and the area does stay the same. Try another point.

(4 , 3) goes to (12 , 3)

Another? (4, 5) goes to (12 , 5)

"It has gone three times bigger."

"The space between two sets of dots is three of those boxes."



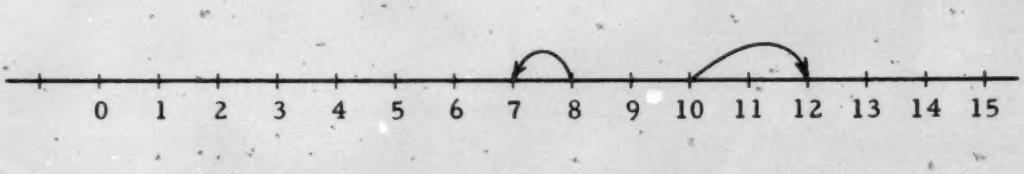
Supplement

Composing Number Line Rules to Move Two Points to Two Points

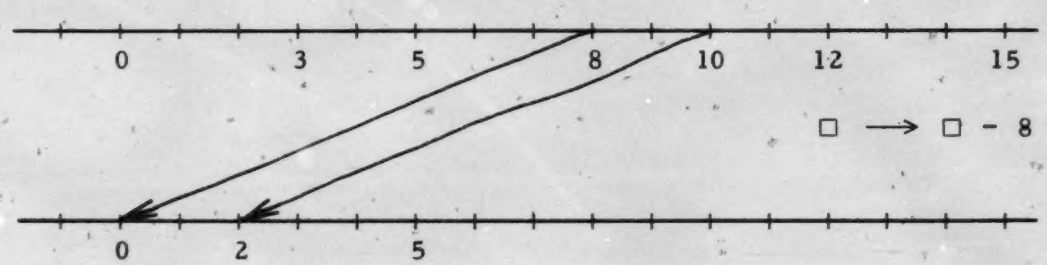
In recent lessons you had to write rules that would send two given starting points to two given landing points.

Another way of looking at the two-points-to-two-points problem is this: We write a sequence of rules each of which does part of the job; we then compose these rules to get a rule that does the whole job.

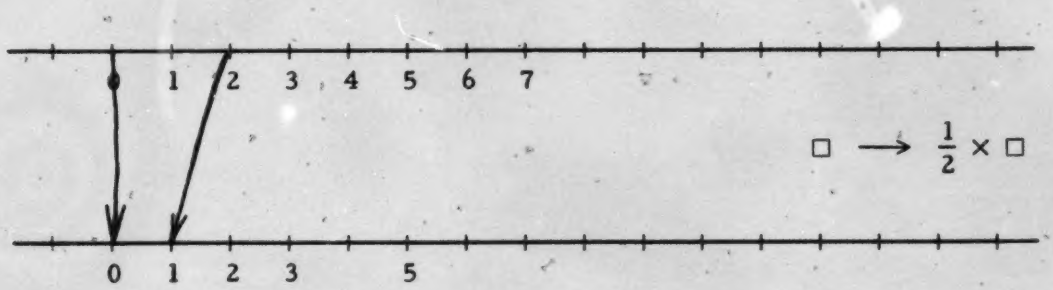
Suppose, for example, that we want a rule which sends $8 \longrightarrow 7$ and $10 \longrightarrow 12$.



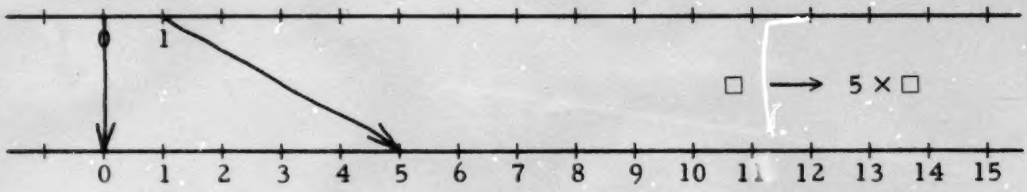
First we write a rule that slides one of the starting points down to 0. The rule $\square \longrightarrow \square - 8$ does this. (Notice that it also sends 10 to 2.) We have:



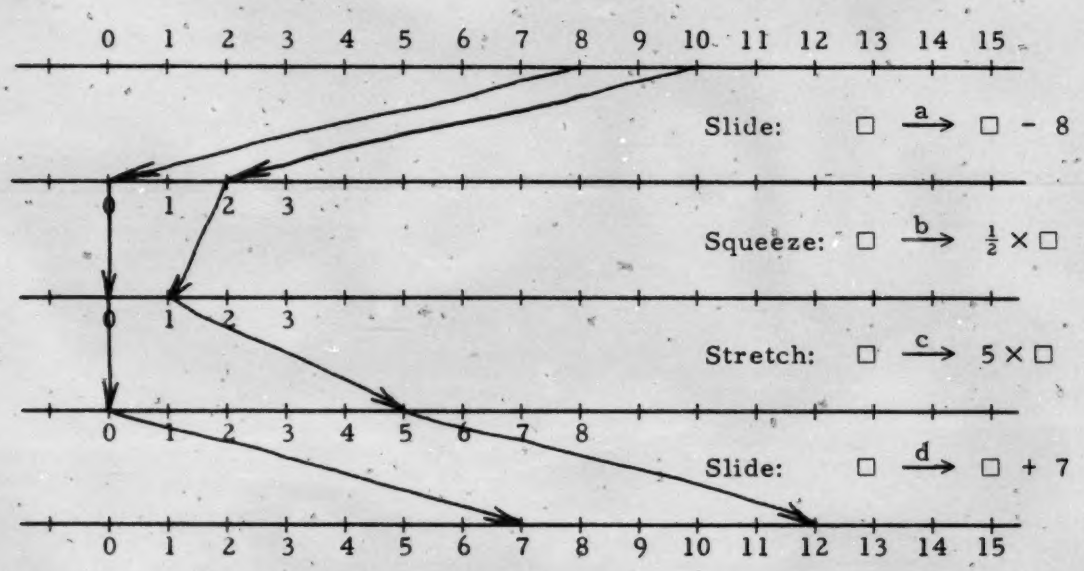
Then we use a rule that sends 2 to 1 but leaves 0 where it is. (We want the two points to be 0 and 1 for a future reason.) A rule that does this is $\square \longrightarrow \frac{1}{2} \times \square$. Now we have:



Then we write a third rule which again keeps 0 fixed but sends 1 to 5. (We choose 5 because the final landing points we want, 7 and 12, are five spaces apart.) The rule $\square \longrightarrow 5 \times \square$ does this. We have:



Finally, we slide 0 and 5 up the line to 7 and 12 by using the rule $\square \longrightarrow \square + 7$. The complete picture looks like this (the rules have been labeled a, b, c, and d):*



The composition of the rules a, b, c and d, namely the rule abcd, is a <u>single</u> rule which, when used <u>once</u>, takes 8 to 7 and 10 to 12. Explicitly, rule abcd is:

$$\square \xrightarrow{\text{abcd}} 5 \times \left(\tfrac{1}{2} \times \left(\square - 8\right)\right) + 7$$

*When you construct such pictures on your own, using graph paper will help you draw suitably aligned number lines.

Before we simplify the rule $\square \xrightarrow{abcd} 5 \times (\frac{1}{2} \times (\square - 8)) + 7$, check it to see that it works:

(1) Start at 8 and use rule abcd once. Land: _____

(2) Start at 10 and use rule abcd once. Land: _____

(3) For which starting place was the computation easier? _____

We can simplify rule abcd from

$$\square \xrightarrow{abcd} 5 \times (\tfrac{1}{2} \times (\square - 8)) + 7$$

to $$\square \xrightarrow{abcd} 5 \times (\tfrac{1}{2} \times \square - 4) + 7$$

or $$\square \xrightarrow{abcd} \tfrac{5}{2} \times \square - 20 + 7$$

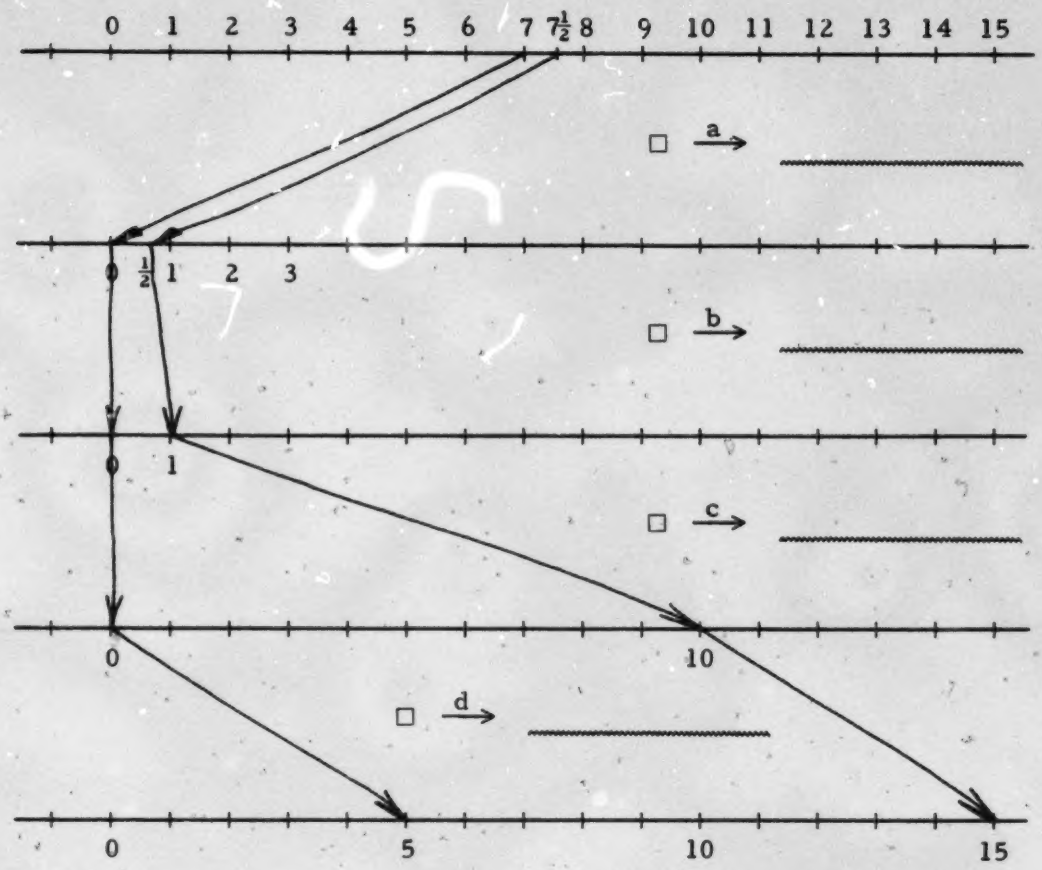
or $$\square \xrightarrow{abcd} 2\tfrac{1}{2} \times \square - 13$$

Some teachers (and some children) prefer this method of doing two-points-to-two-points even though it appears longer, because it shows exactly why $\square \longrightarrow 2\frac{1}{2} \times \square - 13$ works.

In what follows we do <u>not</u> require that you simplify your rules as we did above.

(Answers for this page and the pages following will be found starting on page 28 of this booklet.)

1. Suppose we want a rule that sends 7 to 5 and $7\frac{1}{2}$ to 15.
Complete the rules:

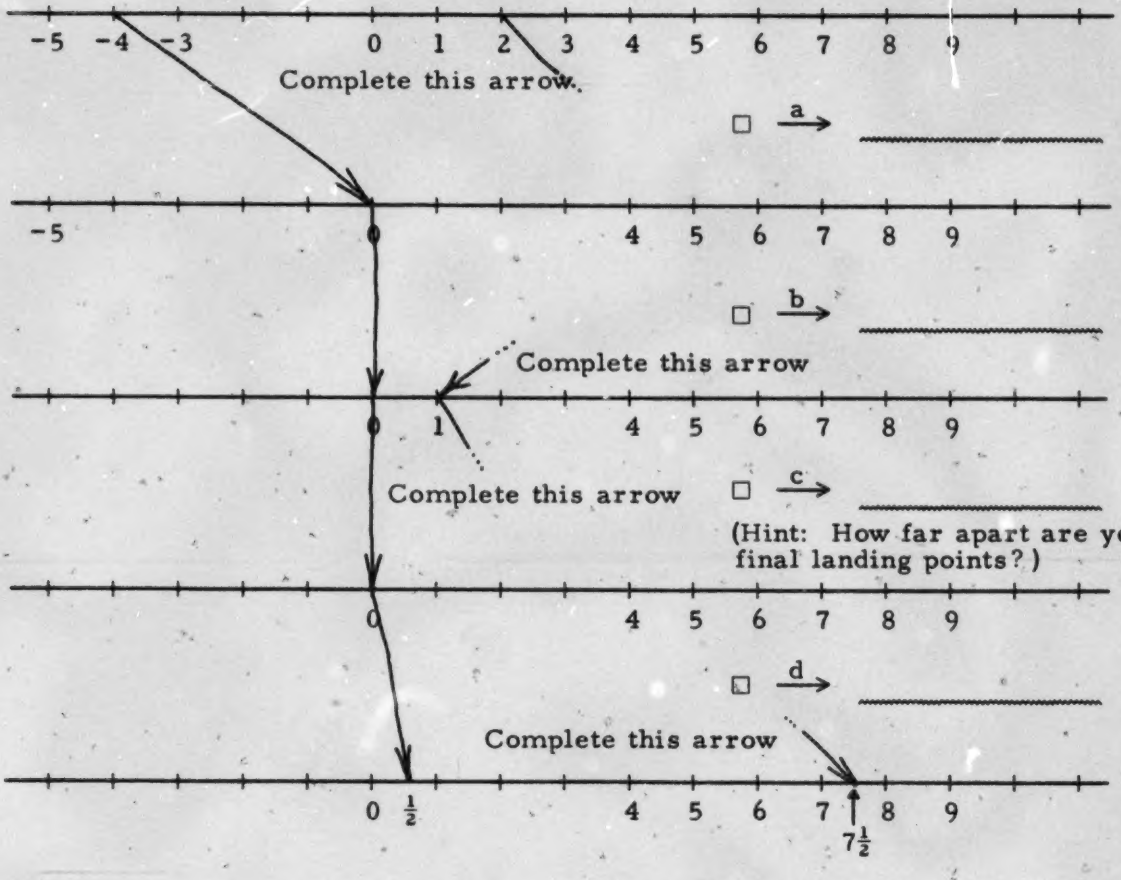


2. Write the composition of rules a , b , c and d .

□ abcd → _____

Check your rule to see that it actually does what it is supposed to do.

3. This time we want $-4 \longrightarrow \frac{1}{2}$ and $2 \longrightarrow 7\frac{1}{2}$.


-5 -4 -3 0 1 2 3 4 5 6 7 8 9
Complete this arrow.
□ a →
-5 0 4 5 6 7 8 9
□ b →
Complete this arrow
0 1 4 5 6 7 8 9
Complete this arrow
□ c →
(Hint: How far apart are your final landing points?)
0 4 5 6 7 8 9
□ d →
Complete this arrow
0 ½ 4 5 6 7 8 9
7½

Write the composition:

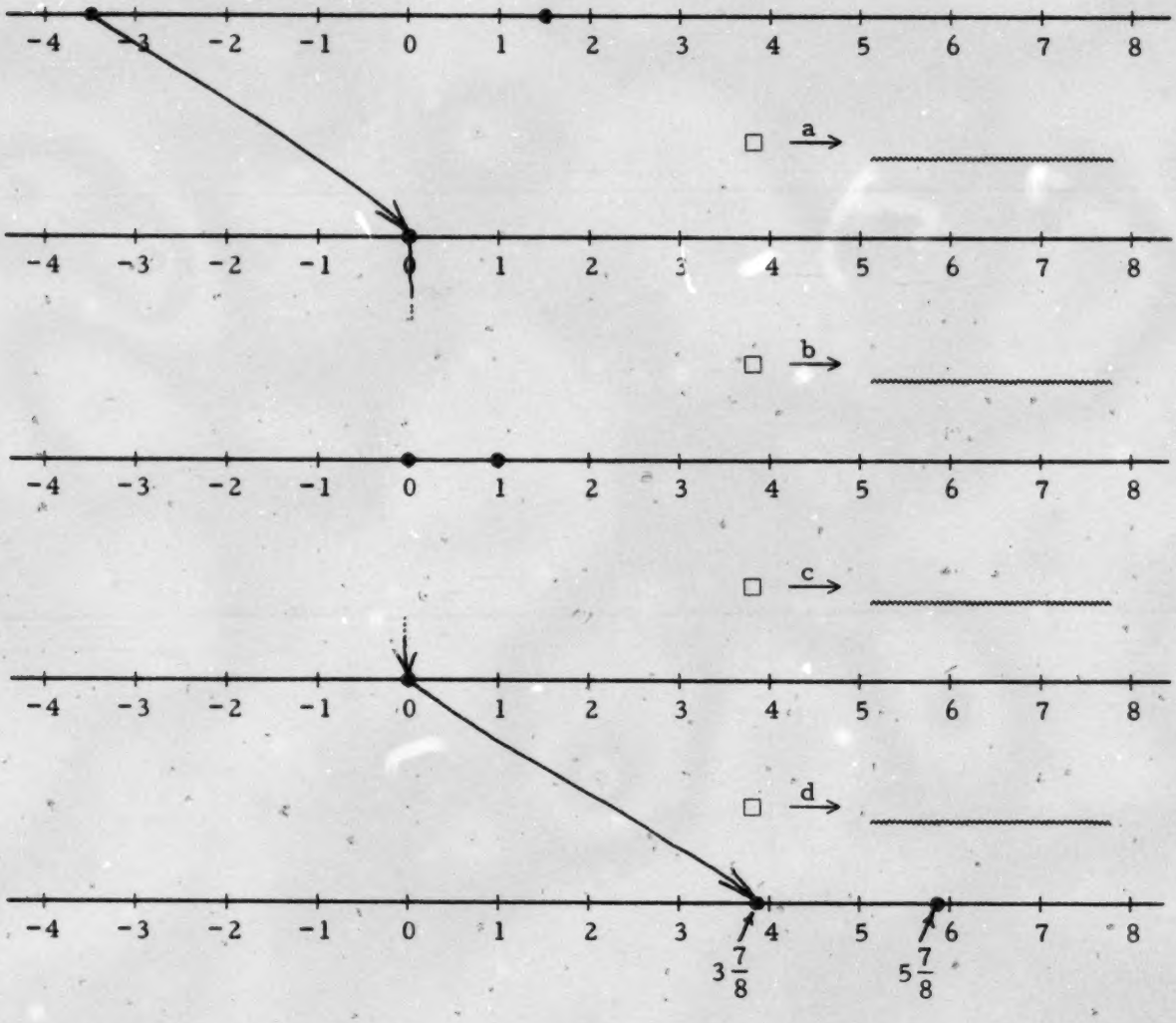
□ abcd → _____

Check your rule starting at -4. Do you land at $\frac{1}{2}$?

Check your rule starting at 2. Do you land at $7\frac{1}{2}$?

4. Now we want $-3\frac{1}{2} \longrightarrow 3\frac{7}{8}$ and $1\frac{1}{2} \longrightarrow 5\frac{7}{8}$.

Draw the missing arrows

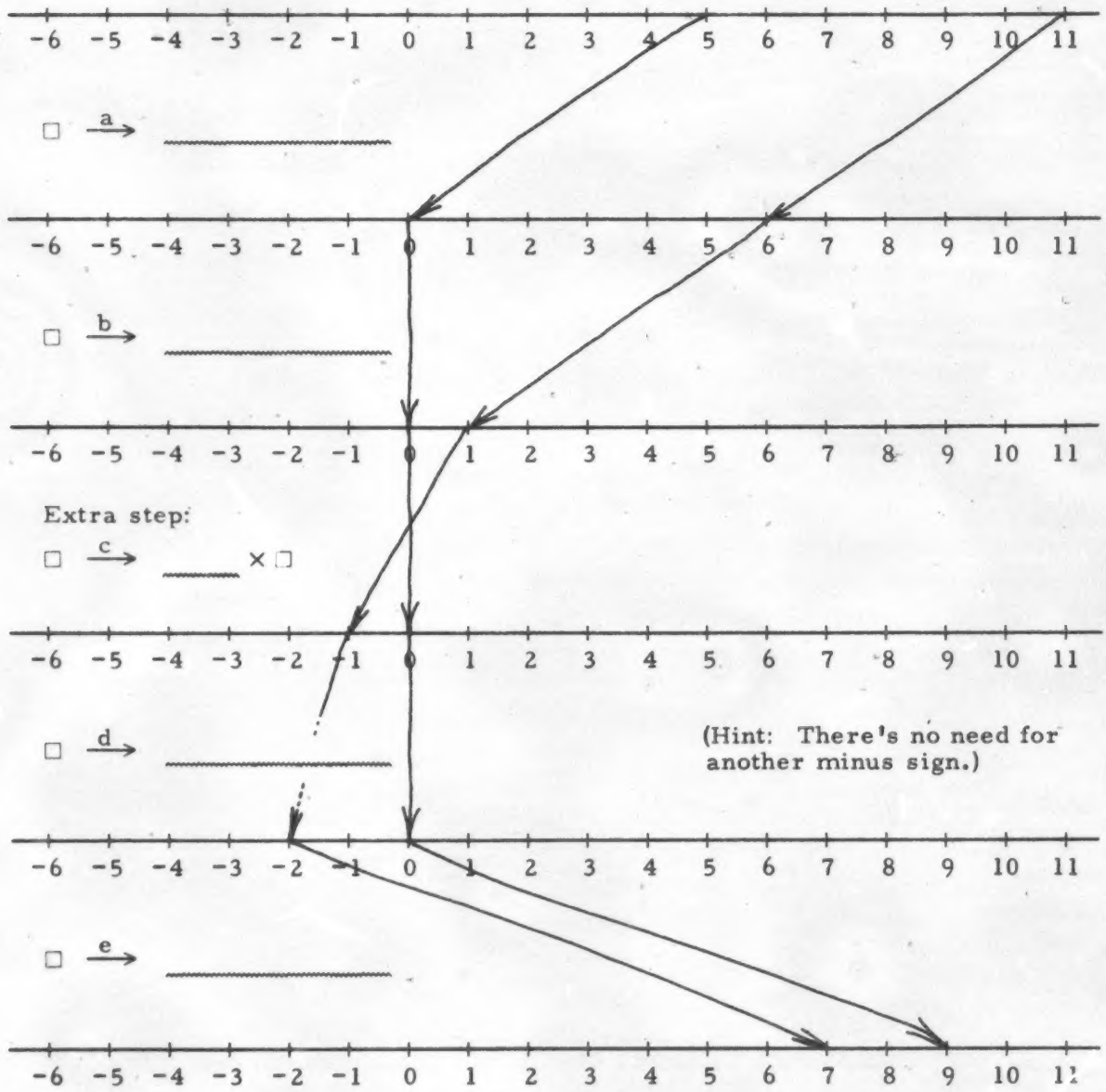


Write the composition:

□ abcd → _____

Can this method be used when the landing points are "switched", that is, when the smaller starting point goes to the larger landing point?

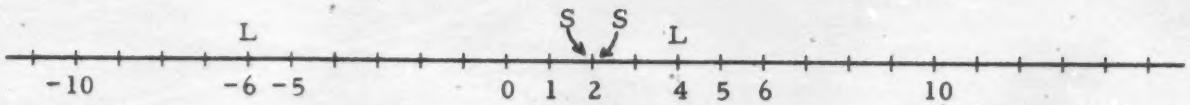
5. Suppose that we want a rule which sends 5 to 9 and 11 to 7. Complete the rules:



Composition:

□ abcde → _____

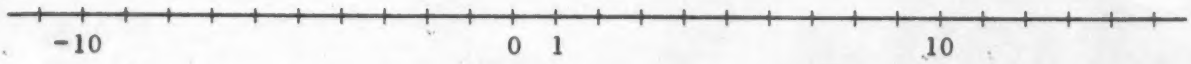
☆6. Try this one: We want $1\frac{19}{20} \longrightarrow 4$ and $2\frac{1}{20} \longrightarrow -6$.



□ $\xrightarrow{a}$ _____

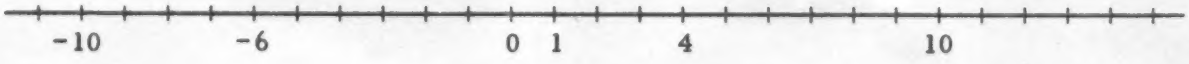
□ $\xrightarrow{b}$ _____

□ $\xrightarrow{c}$ _____



□ $\xrightarrow{d}$ _____

□ $\xrightarrow{e}$ _____



Composition:

□ $\xrightarrow{abcde}$ _____

☆7. On the basis of the clues given below, complete the diagram. You must find both original starting points, both final landing points, and all four intermediate rules.

(i) With each rule the smaller starting point goes to the smaller landing point.

(ii) Each jump with rule a is three spaces long.

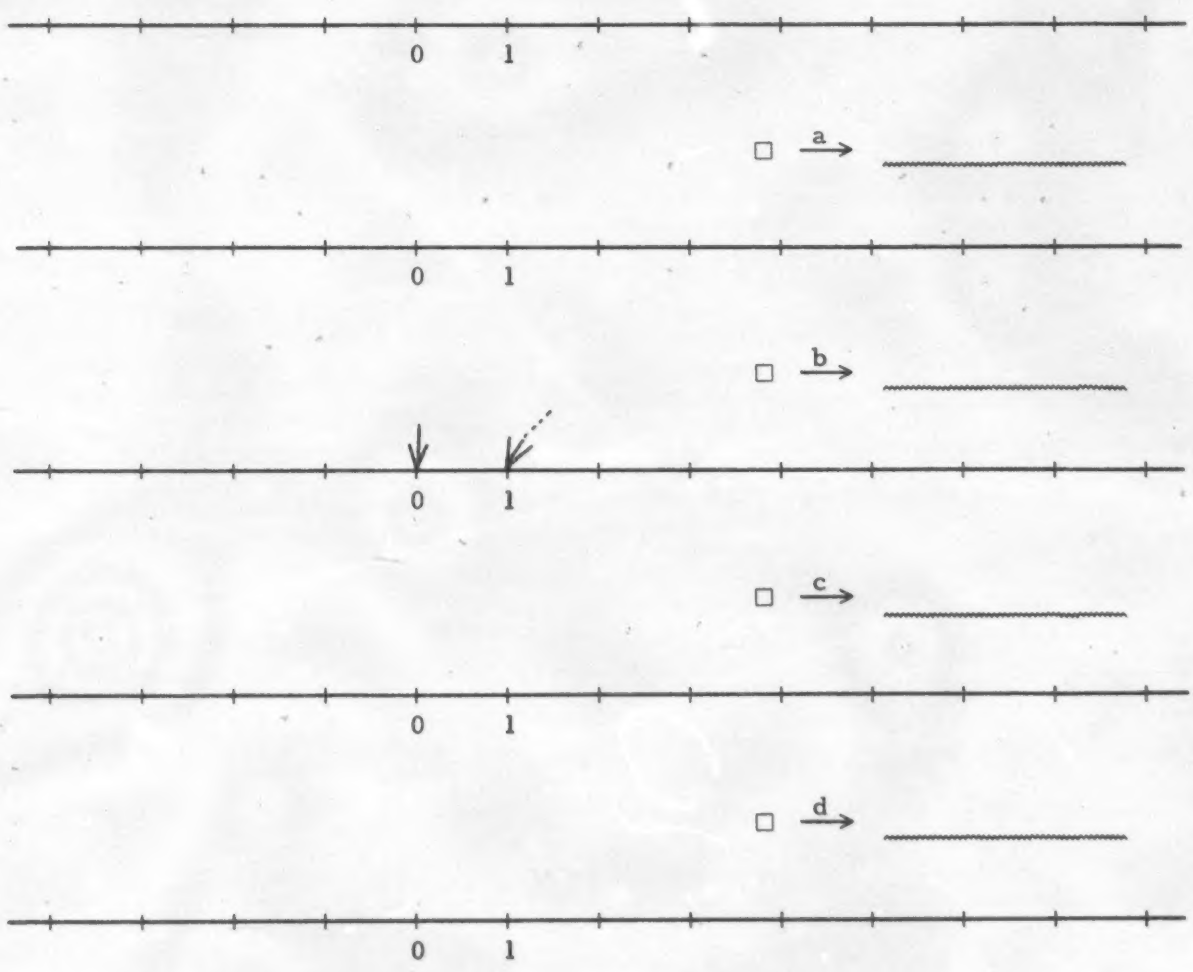
(iii) One of the jumps made with rule b is 7 spaces long.

(iv) One of the jumps made with rule c is $\frac{1}{2}$ space long.

(v) Each jump with rule d is one space long.

(vi) The product of the original starting numbers is less than 20.

(vii) The product of the final landing numbers is negative.



It is natural to ask if we can find rules that send three points to three points or even seven points to seven points. Such rules do exist but they are very complicated and very hard to find. They will involve terms in which many □'s will be multiplied.

Working with problems taking two points to two points gives the student an intuitive understanding of many salient features of linear transformations. He will see these ideas, sometimes in a different garb, arise over and over again.

Before we simplify the rule $\square \xrightarrow{abcd} 5 \times (\frac{1}{2} \times (\square - 8)) + 7$, check it to see that it works:

(1) Start at 8 and use rule abcd once. Land: 7

(2) Start at 10 and use rule abcd once. Land: 12

(3) For which starting place was the computation easier? See note*

We can simplify rule abcd from

$\square \xrightarrow{abcd} 5 \times (\frac{1}{2} \times (\square - 8)) + 7$

to $\square \xrightarrow{abcd} 5 \times (\frac{1}{2} \times \square - 4) + 7$

or $\square \xrightarrow{abcd} \frac{5}{2} \times \square - 20 + 7$

or $\square \xrightarrow{abcd} 2\frac{1}{2} \times \square - 13$

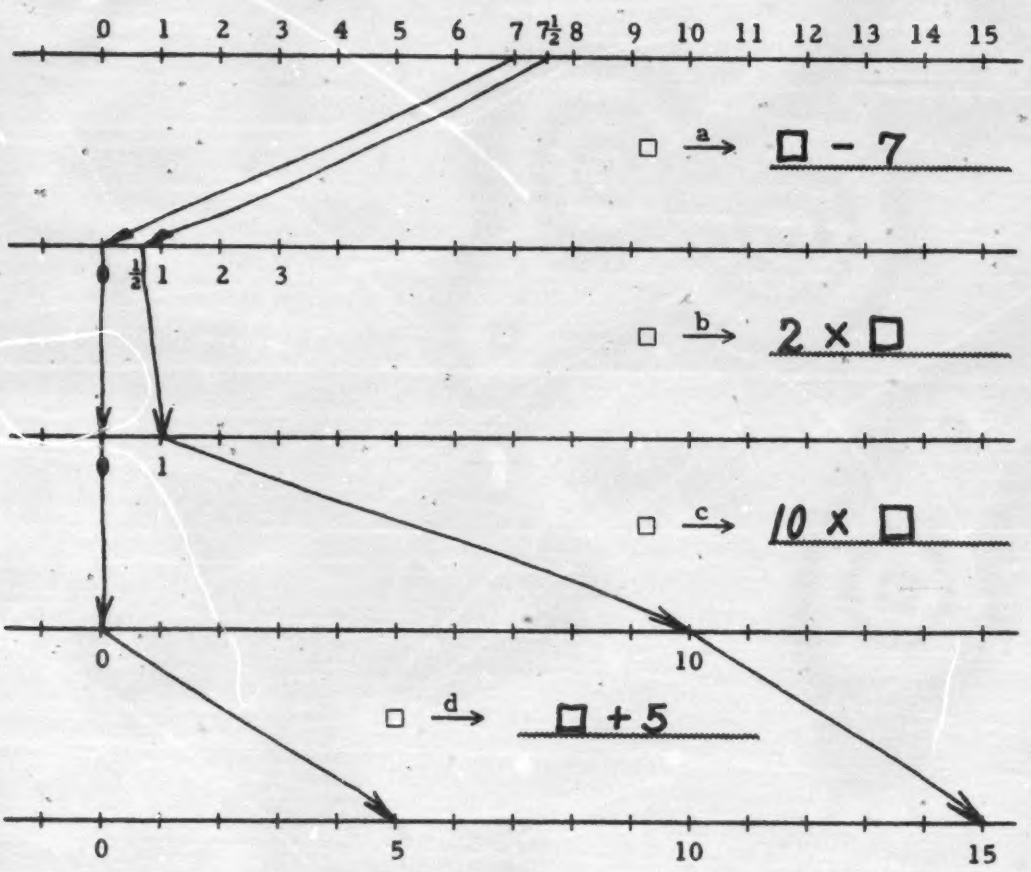
Some teachers (and some children) prefer this method of doing two-points-to-two-points even though it appears longer, because it shows exactly why $\square \longrightarrow 2\frac{1}{2} \times \square - 13$ works.

In what follows we do <u>not</u> require that you simplify your rules as we did above.

---

* Since the question is highly subjective, acceptable answers are
8
10
the same
(Most answers will be 8.)

1. **Suppose we want a rule that sends 7 to 5 and $7\frac{1}{2}$ to 15. Complete the rules:**



2. **Write the composition of rules a , b , c and d .**

$$\square \xrightarrow{abcd} 10 \times (2 \times (\square - 7)) + 5$$

Check your rule to see that it actually does what it is supposed to do.

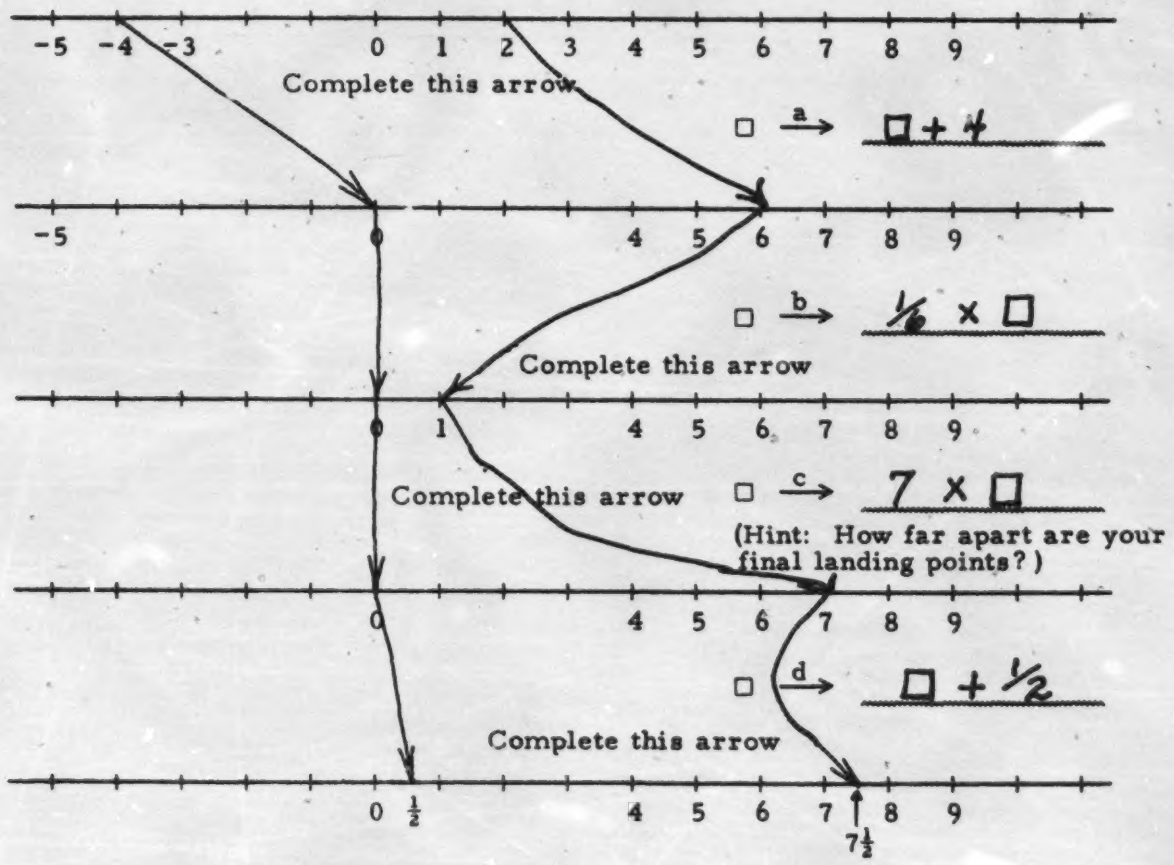
Other ways of writing rule abcd:

$\square \xrightarrow{abcd} 20 \times \square - 135$

$\square \xrightarrow{abcd} 20 \times (\square - 7) + 5$

$\square \xrightarrow{abcd} 2\,(10\,(\square - 7)) + 5$

3. This time we want $-4 \longrightarrow \frac{1}{2}$ and $2 \longrightarrow 7\frac{1}{2}$.


-5 -4 -3 0 1 2 3 4 5 6 7 8 9
Complete this arrow
□ a → □ + 4
-5 0 4 5 6 7 8 9
□ b → 1/6 × □
Complete this arrow
0 1 4 5 6 7 8 9
Complete this arrow
□ c → 7 × □
(Hint: How far apart are your final landing points?)
0 4 5 6 7 8 9
□ d → □ + 1/2
Complete this arrow
0 1/2 4 5 6 7 8 9
7 1/2

Write the composition:

$\square \xrightarrow{abcd} 7 \times \left(\frac{1}{6} \times (\square + 4)\right) + \frac{1}{2}$

Other ways of writing rule abcd:

$\square \xrightarrow{abcd} 1\frac{1}{6} \times \square + 5\frac{1}{6}$

$\square \xrightarrow{abcd} \frac{7}{6}\square + \frac{31}{6}$

$\square \xrightarrow{abcd} \frac{7}{6} \times \square + 4\frac{2}{3} + \frac{1}{2}$

$\square \xrightarrow{abcd} \frac{7 \times \square + 31}{6}$

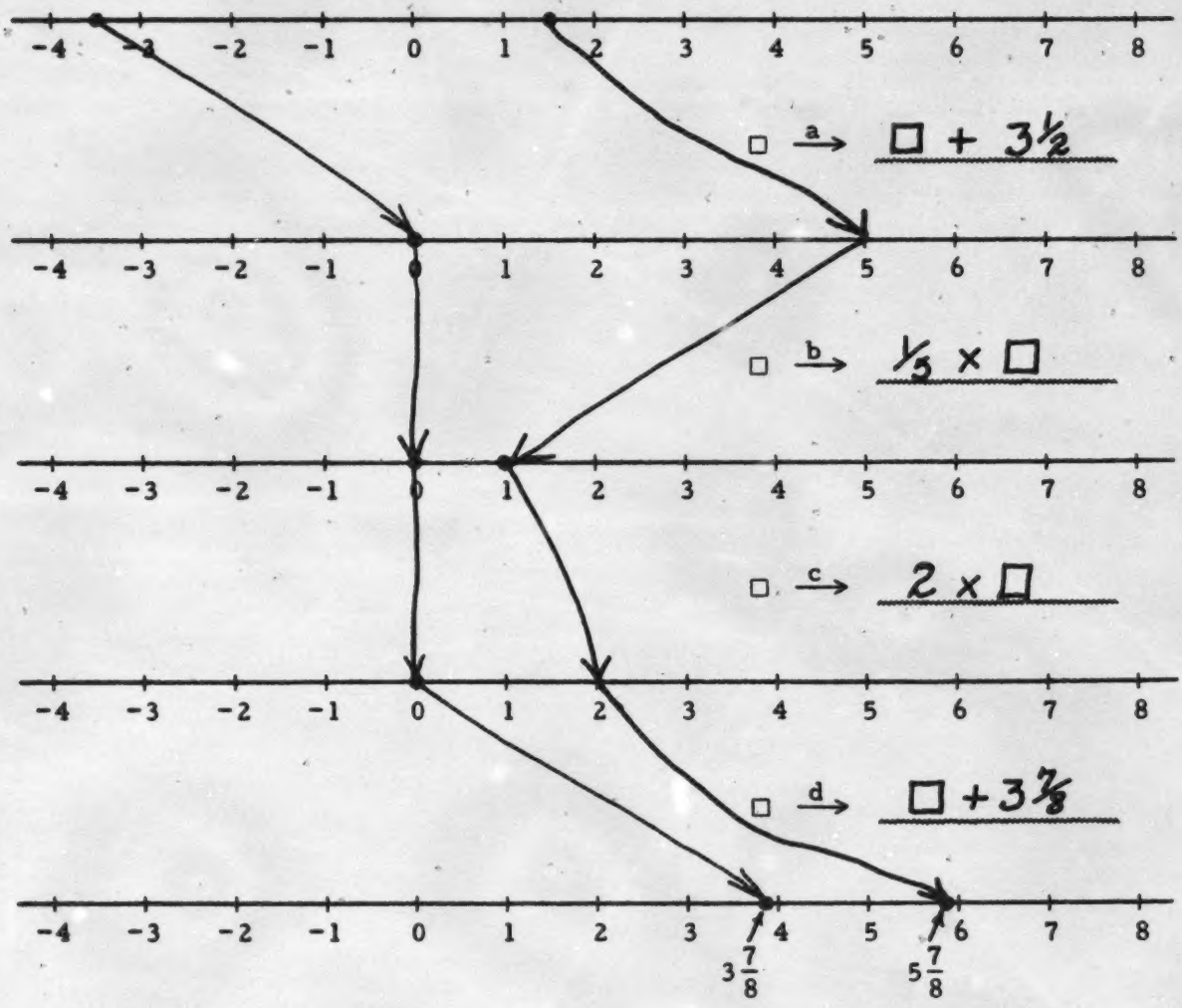
$\square \xrightarrow{abcd} \frac{7}{6}(\square + 4) + \frac{1}{2}$

$\square \xrightarrow{abcd} 7 \times \left(\frac{\square + 4}{6}\right) + \frac{1}{2}$

$\square \xrightarrow{abcd} 7 \times ((\square + 4) \div 6) + \frac{1}{2}$

4. Now we want $-3\frac{1}{2} \longrightarrow 3\frac{7}{8}$ and $1\frac{1}{2} \longrightarrow 5\frac{7}{8}$.

Draw the missing arrows



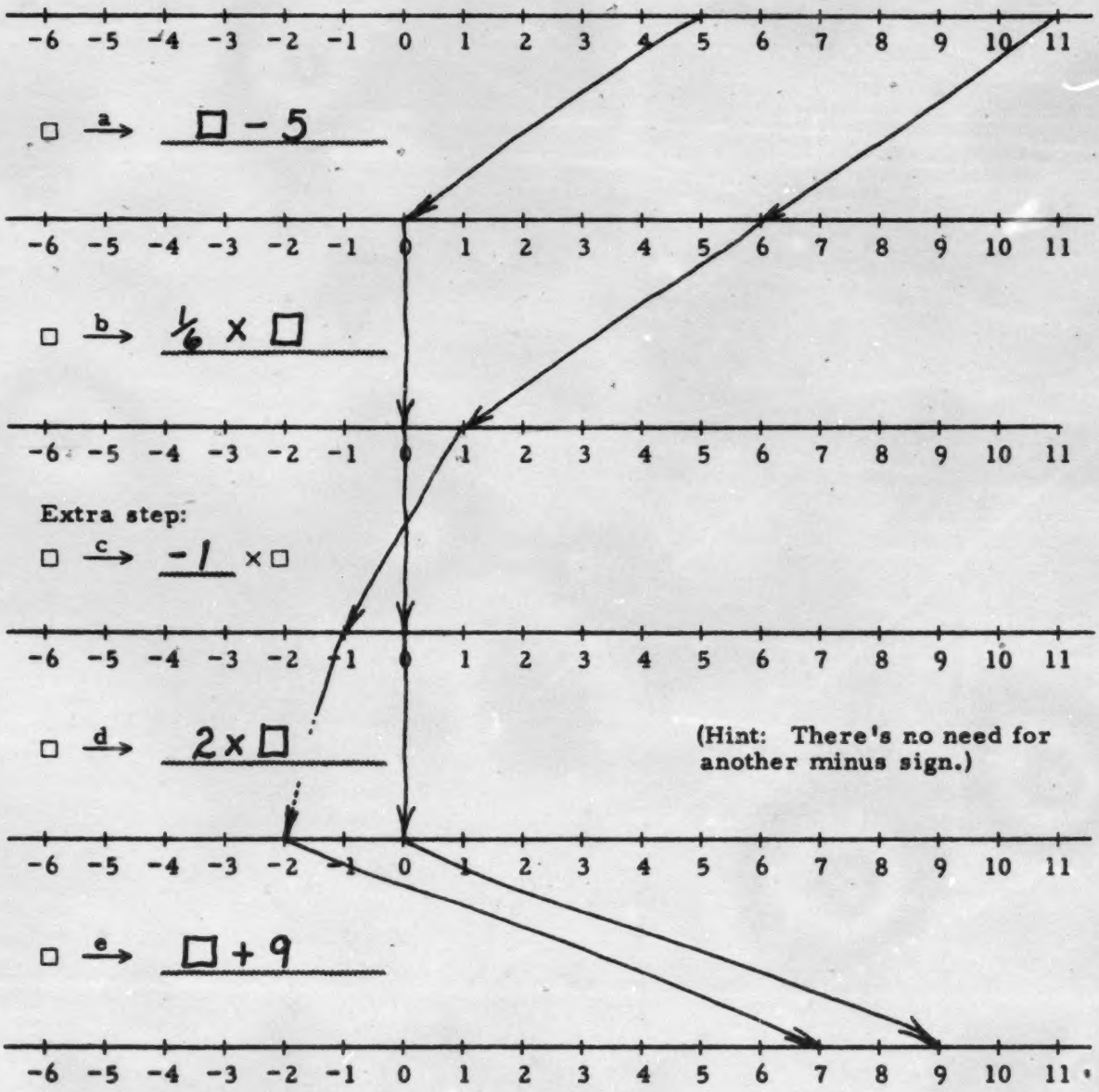
Write the composition:

□ abcd→ $2 \times (\frac{1}{3} \times (\square + 3\frac{1}{2})) + 3\frac{7}{8}$

For additional ways to write this rule, see page 35.

Can this method be used when the landing points are "switched", that is, when the smaller starting point goes to the larger landing point?

5. **Suppose that we want a rule which sends 5 to 9 and 11 to 7. Complete the rules:**

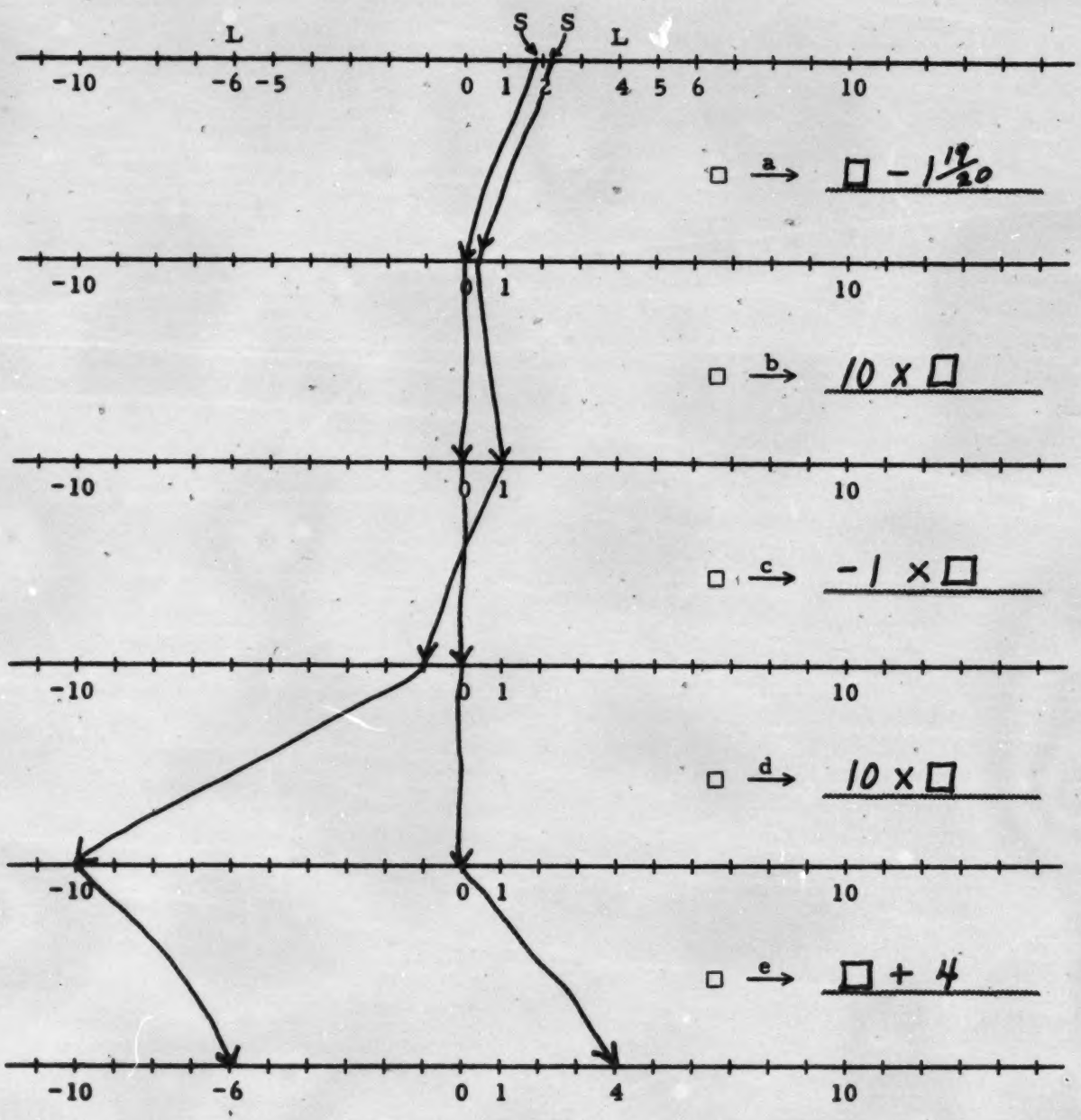


Composition:

□ abcde→ $2\times(-1\times(\frac{1}{6}\times(\square-5)))+9$

For additional ways to write this rule, see page 35.

☆6. **Try this one: We want** $1\frac{19}{20} \longrightarrow 4$ **and** $2\frac{1}{20} \longrightarrow -6$.



Composition:

□ $\xrightarrow{abcde}$ $10\times(-1\times(10\times(\square-1\frac{19}{20})))+4$

For additional ways to write this rule, see page 36.

☆7. **On the basis of the clues given below, complete the diagram. You must find both original starting points, both final landing points, and all four intermediate rules.**

(i) With each rule the smaller starting point goes to the smaller landing point.

(ii) Each jump with rule a is three spaces long.

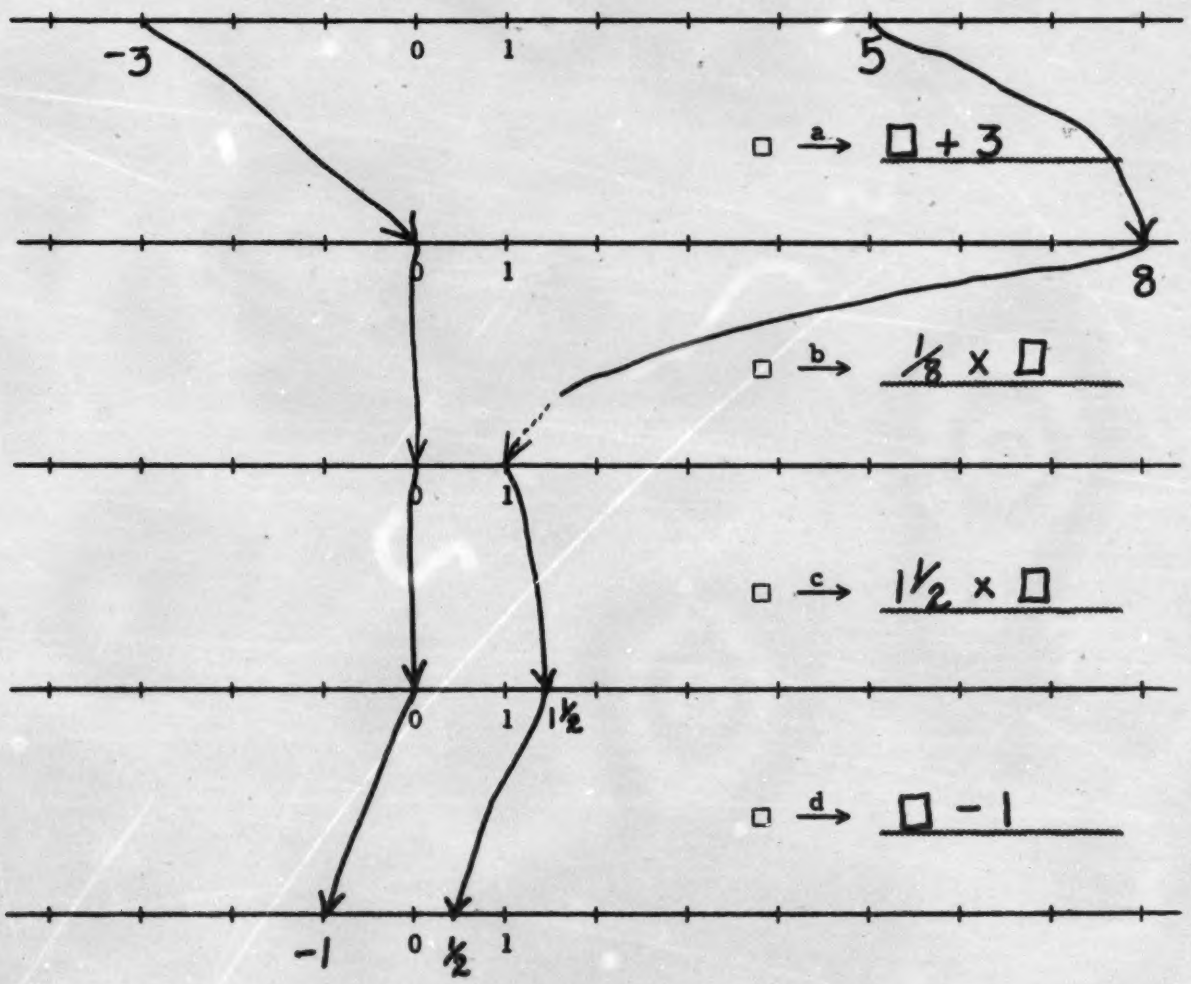
(iii) One of the jumps made with rule b is 7 spaces long.

(iv) One of the jumps made with rule c is $\frac{1}{2}$ space long.

(v) Each jump with rule d is one space long.

(vi) The product of the original starting numbers is less than 20.

(vii) The product of the final landing numbers is negative.



Rule abcd in problem 4 may also be written:

$$\square \xrightarrow{abcd} \frac{2}{5} \times (\square + 3\frac{1}{2}) + 3\frac{7}{8}$$

$$\square \xrightarrow{abcd} 2 \times \frac{(\square + 3\frac{1}{2})}{5} + 3\frac{7}{8}$$

$$\square \xrightarrow{abcd} 2 \times ((\square + 3\frac{1}{2}) \div 5) + 3\frac{7}{8}$$

$$\square \xrightarrow{abcd} \frac{2(\square + \frac{7}{2})}{5} + 3\frac{7}{8}$$

$$\square \xrightarrow{abcd} \frac{2}{5} \times \square + \frac{14}{10} + 3\frac{7}{8}$$

$$\square \xrightarrow{abcd} \frac{2}{5}\square + \frac{7}{5} + 3\frac{7}{8}$$

$$\square \xrightarrow{abcd} \frac{2}{5} \times \square + 1\frac{2}{5} + 3\frac{7}{8}$$

$$\square \xrightarrow{abcd} \frac{2}{5}\square + \frac{211}{40}$$

$$\square \xrightarrow{abcd} \frac{2}{5} \times \square + 5\frac{11}{40}$$

Rule abcde in problem 5 may also be written:

$$\square \xrightarrow{abcde} \frac{1}{6} \times (-1 \times 2 \times (\square - 5)) + 9$$

$$\square \xrightarrow{abcde} -2 \times (\frac{1}{6} \times (\square - 5)) + 9$$

$$\square \xrightarrow{abcde} -2 \times ((\square - 5) \div 6) + 9$$

$$\square \xrightarrow{abcde} -\frac{2}{6}(\square - 5) + 9$$

$$\square \xrightarrow{abcde} -2\left(\frac{\square - 5}{6}\right) + 9$$

$$\square \xrightarrow{abcde} \frac{-\square + 5}{3} + 9$$

$$\square \xrightarrow{abcde} -\frac{1}{3} \times \square + \frac{5}{3} + 9$$

$$\square \xrightarrow{abcde} -\frac{1}{3}\square + 10\frac{2}{3}$$

Rule abcde in problem ☆6 may also be written:

$$\square \xrightarrow{abcde} -1 \times \left(10 \times \left(\square - 1\frac{19}{20} \right) \right) \times 10 + 4$$

$$\square \xrightarrow{abcde} -10 \times 10 \times \left(\square - 1\frac{19}{20} \right) + 4$$

$$\square \xrightarrow{abcde} -10 \left(\frac{\square - 1\frac{19}{20}}{\frac{2}{20}} \right) + 4$$

$$\square \xrightarrow{abcde} -100 \times \left(\square - 1\frac{19}{20} \right) + 4$$

$$\square \xrightarrow{abcde} -100\square + 199$$